21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I TOPIC CATALOGUE AND PREFERENCE GUIDE

MATTHEW ROSENZWEIG

Abstract. This catalogue introduces seven principal and three alternate Round I presentation topics. Browse the individual packets before ranking preferences. Final assignments will balance preference, preparation, presentation order, and mathematical coverage.

1. How to use the materials

Round I is a scaffolded first presentation. No topic requires reading an original research paper. Every assignment asks for a canonical model, a precise principal theorem, at least one complete substantive proof, examples, interpretation, two short exercises, source attribution, and an AI-use disclosure.

First read this catalogue. Then read the *At a glance* and *Why this topic may appeal to you* sections of all seven principal packets. Read the complete packets for your likely top choices and browse the alternates if one better matches your interests. Rank according to genuine interest and fit, not only perceived ease.

2. Principal-topic comparison

No.	Organizing question	Setting	Main proof mechanism	Diff.
01	How do boundary conditions determine the sharp Poincaré constant?	Continuous: an interval and a circle	Fourier series, eigenfunctions, Rayleigh quotients, integration by parts	2/4
02	Why does random walk on a cycle relax on the diffusive time scale?	Discrete: continuous-time simple random walk on $\mathbb{Z}/n\mathbb{Z}$	Finite Fourier analysis, linear algebra, reversible Markov chains	2/4
03	Why does independence permit coordinatewise variance control?	Probability/statistics: functions of independent variables	Conditional variance, Jensen, induction, coordinate resampling	2/4
04	How does a bottleneck force slow convergence to equilibrium?	Discrete geometry: finite connected regular graphs	Rayleigh quotients, level sets, discrete coarea, Cauchy–Schwarz	4/4
05	How does a scale-sensitive inequality produce algebraic decay and smoothing?	Continuous/PDE: the heat equation on the real line	One-dimensional calculus, interpolation, energy estimates, scalar ODEs	3/4
06	Why is the Gaussian Poincaré inequality dimension-free?	Continuous probability: standard Gaussian measure on $\mathbb{R}^d$	Mehler formula, Gaussian integration by parts, semigroup interpolation, Jensen	3/4

Date: Fall 2026.

07	How does a logarithmic Sobolev inequality produce Gaussian concentration?	Probability measures satisfying a logarithmic Sobolev inequality; Gaussian model	Functional entropy, exponential moments, differentiation, Chernoff optimization	4/4

3. Alternate topics

The alternates may replace a principal topic when they better fit student interest or background.

08. **Boolean cube:** Fourier expansion, Poincare inequality, and coordinate influences.
09. **Pinsker inequality:** relative entropy controls total variation and converts entropy decay into probabilistic convergence.
10. **Entropy, likelihood, and Gibbs equilibrium:** relative entropy as both likelihood loss and free-energy excess.

4. What every Round I talk must contain

(1) A precise statement of the model, including all normalizations.
(2) One principal theorem and at least one complete, substantive proof.
(3) A clear distinction between proved statements and inputs.
(4) Examples showing why the theorem is nontrivial and what the constant means.
(5) An interpretation connected to at least one recurring perspective: analytic/dynamical, physical, statistical/information-theoretic, or expository.
(6) Two short exercises, introduced during the talk and supplied with solutions to the instructor. Class solutions will not be collected or graded.
(7) Exact source attribution and an AI-use disclosure.

5. Preference submission

Submit the Round I Topic Preference Google Form linked in the August 28 course announcement by **Sunday, August 30, 2026, at 11:59 p.m. EDT**. The form asks for five distinct ranked choices, at most one topic you strongly prefer not to receive, relevant mathematical background, and genuine scheduling constraints among the anticipated Round I dates. Assignments are not first-come-first-served. Preferences sent by email will not count as a form submission.

6. Difficulty labels

The labels are relative to Round I and reflect the number of new ideas and length of the proof architecture. They do not measure importance, predict a grade, or cap the quality of a presentation. A lower-difficulty topic can support an excellent and mathematically substantial talk.

7. Topic index

01. **Sharp Poincaré inequalities and boundary conditions.** *How do boundary conditions determine the sharp Poincaré constant?*
02. **Random walk on a cycle and diffusive relaxation.** *Why does random walk on a cycle relax on the diffusive time scale?*

03. **Tensorization of variance and the Efron–Stein inequality.** *Why does independence permit coordinatewise variance control?*
04. **Cheeger inequality and bottlenecks on regular graphs.** *How does a bottleneck force slow convergence to equilibrium?*
05. **The Nash inequality and heat-flow smoothing.** *How does a scale-sensitive inequality produce algebraic decay and smoothing?*
06. **Gaussian Poincaré inequality from the Ornstein–Uhlenbeck semigroup.** *Why is the Gaussian Poincaré inequality dimension-free?*
07. **Logarithmic Sobolev inequalities and the Herbst argument.** *How does a logarithmic Sobolev inequality produce Gaussian concentration?*
08. **Fourier analysis, Poincaré inequality, and influences on the Boolean cube.** *How does Fourier analysis convert coordinate sensitivity into a variance bound?*
09. **Pinsker inequality and entropy-to-total-variation convergence.** *How does relative entropy control probabilistic distinguishability?*
10. **Entropy, likelihood, and Gibbs equilibrium.** *Why does relative entropy appear in both statistical inference and equilibrium statistical mechanics?*

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I PRINCIPAL TOPIC PACKET 01 SHARP POINCARÉ INEQUALITIES AND BOUNDARY CONDITIONS

MATTHEW ROSENZWEIG

Abstract. **Organizing question:** How do boundary conditions determine the sharp Poincaré constant? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Principal Round I topic
Setting	Continuous: an interval and a circle
Primary tools	Fourier series, eigenfunctions, Rayleigh quotients, integration by parts
Background	Single-variable analysis; elementary ODEs; basic Fourier series helpful
Relative difficulty	2/4
Principal perspective	Analysis/PDE, with spectral and dynamical interpretations

1. Why this topic may appeal to you

This is the most concrete continuous topic in the slate. It starts from calculus and Fourier series, but already exhibits three recurring principles: the null space depends on the model, the best constant is an eigenvalue, and coercivity becomes exponential relaxation under heat flow.

2. Fixed normalization

Let $L > 0$. Work first with real-valued smooth functions and discuss extension to the corresponding H^1 spaces only at the theorem-map level.

(1) **Dirichlet interval:** $f(0) = f(L) = 0$.
(2) **Mean-zero interval:** $f \in C^1([0, L])$ and $\int_0^L f(x)\, \mathrm{d}x = 0$.
(3) **Periodic circle of length L:** f is L-periodic and $\int_0^L f(x)\, \mathrm{d}x = 0$.

Date: Fall 2026.

3. Principal theorem

Establish the sharp inequalities

$$\int_0^L |f|^2 \, \mathrm{d}x \le \left(\frac{L}{\pi}\right)^2 \int_0^L |f'|^2 \, \mathrm{d}x \qquad \text{(Dirichlet)},$$
$$\int_0^L |f|^2 \, \mathrm{d}x \le \left(\frac{L}{\pi}\right)^2 \int_0^L |f'|^2 \, \mathrm{d}x \qquad \text{(mean-zero interval)},$$
$$\int_0^L |f|^2 \, \mathrm{d}x \le \left(\frac{L}{2\pi}\right)^2 \int_0^L |f'|^2 \, \mathrm{d}x \qquad \text{(periodic mean-zero)}.$$

Identify the equality modes:

$$\sin(\pi x/L), \qquad \cos(\pi x/L), \qquad \sin(2\pi x/L),\ \cos(2\pi x/L),$$

respectively.

4. Required proof content

(1) Give a complete Fourier/eigenfunction proof in at least two settings. Compute the derivative coefficients and state the Parseval identity being used.
(2) Explain the Rayleigh-quotient interpretation: the reciprocal of the sharp Poincaré constant is the smallest admissible eigenvalue of $-\frac{\mathrm{d}^2}{\mathrm{d}x^2}$ once the zero-energy modes are removed.
(3) Derive the associated heat-flow decay. If $u_t = u_{xx}$ and the relevant equilibrium mode has been removed, show

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_0^L |u|^2 \, \mathrm{d}x = -2 \int_0^L |u_x|^2 \, \mathrm{d}x \le -2\lambda_1 \int_0^L |u|^2 \, \mathrm{d}x.$$

(4) Explain the differing constants through the lowest admissible frequency, not merely by quoting formulas.

5. What may be taken as input

You may take completeness of the standard sine, cosine, and complex Fourier bases as known. You should not take the sharp constants or equality cases as inputs.

6. Examples and exercises

Include a function proving sharpness in each setting and a function admissible in one setting but not another. Prepare two exercises, for example: rescale from $[0, 1]$ to $[0, L]$; determine the sharp constant in an even or odd subspace; or deduce heat-flow decay.

7. Common pitfalls

Do not conflate a boundary condition with the mean-zero constraint. Do not assert sharpness without an equality mode. When integrating by parts, state exactly why the boundary contribution vanishes.

8. Suggested reading route and attribution

(1) T. H. Colding, notes prepared by D. Winter, *Introduction to Partial Differential Equations*, MIT OpenCourseWare 18.152: Lecture 4, "The Poincare Inequalities," and Lecture 13, "The Heat Equation." The course page identifies the notes as undergraduate and freely available (MIT OpenCourseWare lecture-notes page).

(2) D. Bakry, I. Gentil, and M. Ledoux, *Analysis and Geometry of Markov Diffusion Operators*, Springer, 2014, the chapter "Poincare Inequalities," pp. 177–233. DOI and publisher page.

The first source provides undergraduate background. The sharp one-dimensional comparison and exact normalization in this assignment must be reconstructed from the Fourier calculation rather than copied from a general theorem.

9. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

10. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY
ROUND I PRINCIPAL TOPIC PACKET 02
RANDOM WALK ON A CYCLE AND DIFFUSIVE RELAXATION

MATTHEW ROSENZWEIG

Abstract. **Organizing question:** Why does random walk on a cycle relax on the diffusive time scale? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Principal Round I topic
Setting	Discrete: continuous-time simple random walk on $\mathbb{Z}/n\mathbb{Z}$
Primary tools	Finite Fourier analysis, linear algebra, reversible Markov chains
Background	Finite probability; complex numbers; eigenvalues and eigenvectors
Relative difficulty	2/4
Principal perspective	Discrete probability and dynamics

1. Why this topic may appeal to you

This is the discrete counterpart of the first topic. It gives an exact calculation rather than an abstract estimate and shows how geometry, Fourier modes, a spectral gap, and a physical relaxation time encode the same phenomenon.

2. Fixed model and normalization

Let $C_n = \mathbb{Z}/n\mathbb{Z}$ and let π be uniform. Use the continuous-time generator

$$Lf(x) = \frac{1}{2}\big(f(x+1) + f(x-1) - 2f(x)\big).$$

The Dirichlet form is

$$\mathcal{E}(f,f) := -\langle f, Lf\rangle_{L^2(\pi)} = \frac{1}{2n}\sum_{x\in C_n}\big(f(x+1) - f(x)\big)^2.$$

This normalization is fixed.

Date: Fall 2026.

3. Principal theorem

For $k = 0, \dots, n-1$, define $\chi_k(x) = e^{2\pi i k x/n}$. Prove that χ_k is an eigenfunction of $-L$ with eigenvalue

$$\lambda_k = 1 - \cos(2\pi k/n) = 2\sin^2(\pi k/n).$$

Consequently,

$$\lambda_{\text{gap}} = 1 - \cos(2\pi/n) = 2\sin^2(\pi/n) \sim \frac{2\pi^2}{n^2}, \qquad \mathrm{Var}_\pi(f) \le \lambda_{\text{gap}}^{-1}\mathcal{E}(f,f).$$

4. Required proof content

(1) Prove stationarity and reversibility and derive the displayed Dirichlet-form identity.
(2) Establish orthogonality of the finite Fourier characters and diagonalize L completely.
(3) Identify the first nonconstant eigenspace and the equality cases in the Poincaré inequality.
(4) For $P_t = e^{tL}$, prove

$$\mathrm{Var}_\pi(P_t f) \le e^{-2\lambda_{\text{gap}} t}\,\mathrm{Var}_\pi(f).$$

(5) Explain why the characteristic time is of order n^2, connecting the first mode with diffusive spatial scaling.

5. What may be taken as input

You may use the spectral theorem for a real symmetric matrix and $\sin x \sim x$ as $x \to 0$. The Fourier diagonalization itself is required.

6. Examples and exercises

Compare with at least two of: the complete graph, a reflecting path, or a different jump-rate normalization. Prepare two exercises, such as computing C_4 directly, diagonalizing the lazy discrete-time walk, or comparing the first discrete mode with the continuum circle.

7. Common pitfalls

Do not switch between discrete time and continuous time without translating eigenvalues. Keep track of directed versus undirected edge sums. State whether the Fourier calculation is being performed over real- or complex-valued functions.

8. Suggested reading route and attribution

(1) D. A. Levin and Y. Peres, with contributions by E. L. Wilmer, *Markov Chains and Mixing Times*, 2nd ed., AMS, 2017: Chapter 12, "Eigenvalues," and Chapter 20, "Continuous-time chains." Public PDF from the authors.
(2) R. van Handel, *Probability in High Dimension*, APC 550 lecture notes: Sections 2.2–2.4, approximately pp. 21–40, for Markov semigroups, Poincare inequalities, variance identities, and exponential ergodicity (public course notes).

The exact cycle spectrum should be computed in the talk; the references supply the surrounding spectral and semigroup framework.

9. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

10. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I PRINCIPAL TOPIC PACKET 03 TENSORIZATION OF VARIANCE AND THE EFRON–STEIN INEQUALITY

MATTHEW ROSENZWEIG

Abstract. **Organizing question:** Why does independence permit coordinatewise variance control? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Principal Round I topic
Setting	Probability/statistics: functions of independent variables
Primary tools	Conditional variance, Jensen, induction, coordinate resampling
Background	Finite probability; independence; variance
Relative difficulty	2/4
Principal perspective	Probability, statistics, and product structure

1. Why this topic may appeal to you

This is the most directly statistical topic among the principal assignments. It explains why high dimension need not create large fluctuations when coordinates are independent and develops a proof method that recurs throughout concentration theory.

2. Fixed setting

Let $X_1, \ldots, X_n$ be independent random variables and let $f(X_1, \ldots, X_n)$ be square-integrable. For fixed values of all coordinates except i, write $\mathrm{Var}_i\, f$ for variance in the ith coordinate only.

3. Principal theorems

Theorem 1 (Tensorization of variance)**.**

$$\mathrm{Var}(f(X_1, \ldots, X_n)) \leq \sum_{i=1}^{n} \mathbf{E}[\mathrm{Var}_i\, f(X_1, \ldots, X_n)].$$

Let X_i' be an independent copy of X_i and let $X^{(i)}$ replace X_i by X_i'. Deduce

Date: Fall 2026.

Theorem 2 (Efron–Stein)**.**

$$\operatorname{Var}(f(X)) \le \frac{1}{2}\sum_{i=1}^{n} \mathbf{E}\big[(f(X) - f(X^{(i)}))^2\big].$$

4. Required proof content

(1) Prove the finite-space law of total variance

$$\operatorname{Var}(Y) = \mathbf{E}[\operatorname{Var}(Y \mid Z)] + \operatorname{Var}(\mathbf{E}[Y \mid Z])$$

by direct algebra.

(2) Prove tensorization first for two coordinates and then by induction, identifying precisely where independence and Jensen's inequality enter.

(3) Prove the conditional resampling identity

$$\mathbf{E}\big[(f(X) - f(X^{(i)}))^2 \mid X_{-i}\big] = 2\operatorname{Var}_i f.$$

(4) Derive Efron–Stein and explain equality for additive functions $f(x) = \sum_i g_i(x_i)$.

5. Required applications

Treat at least two examples: the empirical average; a coordinatewise sensitive statistic; or a Boolean function. For the empirical average, show explicitly how the correct n^{-1} variance scale appears.

6. What may be taken as input

You may work entirely on finite probability spaces and mention the general square-integrable version as an extension. Do not invoke abstract Hilbert-space conditional expectation without explaining the finite calculation it replaces.

7. Exercises and common pitfalls

Possible exercises include equality for additive functions, a product-space Poincare inequality, or a variance estimate for a bounded statistic. Independence is essential; every conditional expectation must display the variables being held fixed; and the factor 1/2 in Efron–Stein must be derived rather than memorized.

8. Suggested reading route and attribution

(1) R. van Handel, *Probability in High Dimension*, Section 2.1, approximately pp. 13–20, for tensorization and bounded differences (public course notes).

(2) B. Efron and C. Stein, "The jackknife estimate of variance," *Annals of Statistics* 9 (1981), no. 3, 586–596. DOI and journal page. This is a historical source; the presentation should use the elementary modern proof above.

9. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

10. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I PRINCIPAL TOPIC PACKET 04 CHEEGER INEQUALITY AND BOTTLENECKS ON REGULAR GRAPHS

MATTHEW ROSENZWEIG

Abstract. **Organizing question:** How does a bottleneck force slow convergence to equilibrium? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Principal Round I topic
Setting	Discrete geometry: finite connected regular graphs
Primary tools	Rayleigh quotients, level sets, discrete coarea, Cauchy–Schwarz
Background	Graph theory; finite Markov chains; comfort with multi-step proofs
Relative difficulty	4/4
Principal perspective	Discrete geometry, probability, and spectral theory

1. Why this topic may appeal to you

This is the most geometric discrete topic and one of the most demanding Round I choices. It converts a global analytic quantity, the spectral gap, into a statement about cuts and bottlenecks. The proof contains a genuine mechanism rather than only an exact calculation.

2. Fixed normalization

Let $G = (V, E)$ be a finite connected d-regular graph. Let P be the simple-random-walk transition matrix, $L = P - I$, and π uniform. Then

$$\mathcal{E}(f, f) = \frac{1}{d|V|} \sum_{\{x,y\} \in E} (f(x) - f(y))^2.$$

Define

$$\Phi := \min_{\varnothing \neq S \subseteq V,\ |S| \leq |V|/2} \frac{|\partial_E S|}{d|S|},$$

where $\partial_E S$ consists of edges with exactly one endpoint in S. Let λ be the spectral gap of $I - P$.

Date: Fall 2026.

3. Principal theorem

Prove

$$\frac{\Phi^2}{2} \le \lambda \le 2\Phi.$$

The upper estimate says a sparse cut yields a low-energy test function. The lower estimate says a low-energy function has a level set giving a sparse cut.

4. Required proof architecture

(1) **Indicator test:** insert a centered indicator and prove $\lambda \le 2\Phi$.
(2) **Discrete coarea:** for $h \ge 0$, prove

$$\sum_{\{x,y\}\in E} |h(x) - h(y)| = \int_0^\infty |\partial_E\{h > t\}| \,\mathrm{d}t.$$

(3) **One-sided estimate:** if $g \ge 0$ and $|\operatorname{supp} g| \le |V|/2$, apply coarea to g^2 and Cauchy–Schwarz to prove

$$\mathcal{E}(g,g) \ge \frac{\Phi^2}{2} \|g\|^2_{L^2(\pi)} .$$

(4) **Median reduction:** subtract a median, decompose into positive and negative parts, justify the support bounds and energy inequality, and conclude the lower Cheeger estimate.

5. Examples required

Discuss the cycle and complete graph. Also construct or describe a regular dumbbell-type graph with two well-connected pieces joined through few cross-edges and explain why the cut predicts slow relaxation.

6. What may be taken as input

The Rayleigh characterization of the spectral gap may be used. The regular-graph proof above should otherwise be self-contained. A general weighted reversible-chain theorem may be stated only after the proof.

7. Exercises and common pitfalls

Possible exercises: estimate Φ of a cycle; prove the easy direction for a reversible chain; or find a bottleneck test function. Do not mix normalized and unnormalized Laplacians. Conductance and edge expansion differ by degree and stationary-mass factors; every constant must be translated to the fixed normalization.

8. Suggested reading route and attribution

(1) D. A. Levin and Y. Peres, with contributions by E. L. Wilmer, *Markov Chains and Mixing Times*, 2nd ed.: Chapter 7, "Lower bounds on mixing times," and Chapter 13, "Eigenfunctions and comparison of chains," including the Cheeger-type inequalities and historical notes (public PDF from the authors).

(2) J. Cheeger, "A lower bound for the smallest eigenvalue of the Laplacian," in R. C. Gunning, ed., *Problems in Analysis*, Princeton University Press, 1970, pp. 195–199. This is the manifold antecedent; graph and reversible-chain versions are attributed in the historical notes of Levin–Peres.

9. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

10. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I PRINCIPAL TOPIC PACKET 05 THE NASH INEQUALITY AND HEAT-FLOW SMOOTHING

MATTHEW ROSENZWEIG

Abstract. **Organizing question:** How does a scale-sensitive inequality produce algebraic decay and smoothing? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Principal Round I topic
Setting	Continuous/PDE: the heat equation on the real line
Primary tools	One-dimensional calculus, interpolation, energy estimates, scalar ODEs
Background	Single-variable analysis; elementary PDE notation
Relative difficulty	3/4
Principal perspective	Analysis/PDE and scale-sensitive functional inequalities

1. Why this topic may appeal to you

This topic is the cleanest way to see the branch of the course that is not organized around an equilibrium probability measure. It explains why whole-space heat flow decays algebraically rather than exponentially and why scaling determines the exponents.

2. Fixed setting and principal inequalities

Work first with $f \in C_c^\infty(\mathbb{R})$. Prove

$$\|f\|_\infty^2 \le 2\,\|f\|_2\,\|f'\|_2$$

and deduce the one-dimensional Nash inequality

$$\|f\|_2^6 \le 4\,\|f\|_1^4\,\|f'\|_2^2.$$

The constant is secondary; the exponents and scaling are essential.

3. Required heat-flow application

For $u_t = u_{xx}$ on $\mathbb{R}$, set $y(t) = \|u(t)\|_2^2$ and $M = \|u(0)\|_1$. Using L^1 contraction and the energy identity, prove

$$y'(t) = -2\,\|u_x(t)\|_2^2 \le -\frac{y(t)^3}{2M^4}.$$

Date: Fall 2026.

Solve the differential inequality and obtain

$$\|u(t)\|_2 \le Mt^{-1/4}, \qquad t > 0,$$

up to the harmless constant dictated by your normalization. State the resulting $L^1 \to L^2$ operator estimate.

4. Required proof content

(1) Prove the L^∞ estimate from the fundamental theorem of calculus applied to f^2.
(2) Derive Nash using $\|f\|_2^2 \le \|f\|_1 \|f\|_\infty$.
(3) Check every scaling exponent under $f_a(x) = f(ax)$ and use the same scaling to explain the failure of a whole-line Poincare inequality with a fixed constant.
(4) Derive the heat energy identity and solve the scalar differential inequality without skipping the algebra.
(5) Compare algebraic Nash decay on $\mathbb{R}$ with exponential Poincare decay on a compact interval or circle.

5. What may be taken as input

You may take L^1 contraction of the heat semigroup as an input, but explain it through the heat kernel and Young's inequality or through positivity and mass preservation. The explicit heat kernel is a check, not a replacement for the Nash proof.

6. Optional extension, exercises, and pitfalls

Using self-adjointness and the semigroup property, derive an $L^1 \to L^\infty$ bound of order $t^{-1/2}$. Exercises may address the scalar ODE, dimension-d scaling, or this semigroup step. Distinguish decay of a norm from decay of its square; do not conceal the use of L^1 contraction; and do not call Nash a Poincare inequality.

7. Suggested reading route and attribution

(1) T. H. Colding, notes prepared by D. Winter, *Introduction to Partial Differential Equations*, MIT OpenCourseWare 18.152, Lecture 13, "The Heat Equation," for undergraduate heat-equation background (course page).
(2) J. Nash, "Continuity of solutions of parabolic and elliptic equations," *American Journal of Mathematics* 80 (1958), no. 4, 931–954. DOI and journal record. This is the historical source; the talk uses a much simpler one-dimensional route.
(3) D. Bakry, I. Gentil, and M. Ledoux, *Analysis and Geometry of Markov Diffusion Operators*, the chapter "Sobolev Inequalities," pp. 277–343, for the general relation among Sobolev/Nash inequalities, semigroup bounds, and heat kernels. DOI and publisher page.

8. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

9. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I PRINCIPAL TOPIC PACKET 06 GAUSSIAN POINCARÉ INEQUALITY FROM THE ORNSTEIN–UHLENBECK SEMIGROUP

MATTHEW ROSENZWEIG

Abstract. **Organizing question:** Why is the Gaussian Poincaré inequality dimension-free? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Principal Round I topic
Setting	Continuous probability: standard Gaussian measure on $\mathbb{R}^d$
Primary tools	Mehler formula, Gaussian integration by parts, semigroup interpolation, Jensen
Background	Multivariable calculus; Gaussian random vectors; basic semigroup language
Relative difficulty	3/4
Principal perspective	Probability, dynamics, and high-dimensional analysis

1. Why this topic may appeal to you

This is the first genuinely high-dimensional continuous model. It gives a complete semigroup proof, exhibits several reusable identities, and explains why independent Gaussian coordinates do not force deterioration of the Poincare constant.

2. Fixed normalization

Let γ_d be standard Gaussian measure on $\mathbb{R}^d$ and define

$$P_t f(x) = \mathbf{E}\left[f\left(e^{-t}x + \sqrt{1-e^{-2t}}\,G\right)\right], \qquad G \sim \gamma_d.$$

The generator is $L = \Delta - x \cdot \nabla$.

3. Principal theorem

For smooth f with sufficient growth control,

$$\mathrm{Var}_{\gamma_d}(f) \le \int_{\mathbb{R}^d} |\nabla f|^2 \, \mathrm{d}\gamma_d.$$

The constant 1 is sharp, independent of d, and attained by nonconstant affine functions.

Date: Fall 2026.

4. Required proof content

(1) Prove Gaussian invariance of the Mehler formula.
(2) Prove the commutation identity $\nabla P_t f = e^{-t} P_t(\nabla f)$.
(3) Establish Gaussian integration by parts and deduce

$$\frac{\mathrm{d}}{\mathrm{d}t} \int (P_t f)^2 \, \mathrm{d}\gamma_d = -2 \int |\nabla P_t f|^2 \, \mathrm{d}\gamma_d.$$

(4) Show $P_t f \to \int f \, \mathrm{d}\gamma_d$ and derive

$$\mathrm{Var}_{\gamma_d}(f) = 2 \int_0^\infty \int |\nabla P_t f|^2 \, \mathrm{d}\gamma_d \, \mathrm{d}t.$$

(5) Use Jensen and invariance to prove

$$\int |\nabla P_t f|^2 \, \mathrm{d}\gamma_d \le e^{-2t} \int |\nabla f|^2 \, \mathrm{d}\gamma_d,$$

then integrate.

5. Interpretation and comparison

Explain exponential contraction of nonconstant observables under Ornstein–Uhlenbeck evolution and the meaning of a dimension-free constant. Compare this proof conceptually with tensorization. Dimension-free does not mean every variance is bounded independently of d: the gradient energy may grow with d.

6. What may be taken as input

You may use the semigroup property after checking it from Gaussian covariance. Approximation beyond smooth bounded functions may be described rather than proved. The five identities above are the required proof.

7. Exercises and common pitfalls

Possible exercises include Gaussian integration by parts in one dimension, equality for affine functions, or exponential variance decay. Do not write $P_t(|\nabla f|)^2 \le P_t(|\nabla f|^2)$ without identifying Jensen's inequality. Keep the generator and Mehler normalizations consistent.

8. Suggested reading route and attribution

(1) D. Chafaï and J. Lehec, *Logarithmic Sobolev Inequalities Essentials*, Sections 1.2–1.4, approximately pp. 8–17, for the Ornstein–Uhlenbeck semigroup, Gaussian Poincare inequality, and convergence to equilibrium (public lecture notes).
(2) R. van Handel, *Probability in High Dimension*, Sections 2.2–2.4, approximately pp. 21–40 (public course notes).
(3) D. Bakry, I. Gentil, and M. Ledoux, *Analysis and Geometry of Markov Diffusion Operators*, pp. 177–233, as an instructor-level reference. DOI and publisher page.

9. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

10. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I PRINCIPAL TOPIC PACKET 07 LOGARITHMIC SOBOLEV INEQUALITIES AND THE HERBST ARGUMENT

MATTHEW ROSENZWEIG

ABSTRACT. **Organizing question:** How does a logarithmic Sobolev inequality produce Gaussian concentration? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Principal Round I topic
Setting	Probability measures satisfying a logarithmic Sobolev inequality; Gaussian model
Primary tools	Functional entropy, exponential moments, differentiation, Chernoff optimization
Background	Calculus; probability; comfort manipulating expectations
Relative difficulty	4/4
Principal perspective	Information theory, probability, and concentration

1. Why this topic may appeal to you

This topic gives one of the central implication chains in modern probability: a local energy inequality controls exponential moments and hence rare fluctuations. It has the strongest information-theoretic flavor among the principal topics.

2. Fixed functional inequality

Assume a probability measure μ on $\mathbb{R}^d$ satisfies

$$\mathrm{Ent}_\mu(e^g) \leq \frac{C}{2} \int e^g |\nabla g|^2 \, \mathrm{d}\mu$$

for smooth g, where

$$\mathrm{Ent}_\mu(h) = \int h \log h \, \mathrm{d}\mu - \left(\int h \, \mathrm{d}\mu \right) \log \left(\int h \, \mathrm{d}\mu \right).$$

For standard Gaussian measure, $C = 1$ in this normalization.

Date: Fall 2026.

3. Principal theorem

If F is L-Lipschitz, prove

$$\log \int e^{\lambda(F-\int F \, \mathrm{d}\mu)} \, \mathrm{d}\mu \le \frac{CL^2\lambda^2}{2}, \qquad \lambda \in \mathbb{R},$$

and hence

$$\mu\left(F - \int F \, \mathrm{d}\mu \ge r\right) \le \exp\left(-\frac{r^2}{2CL^2}\right).$$

Apply the same argument to $-F$ for the two-sided bound.

4. Required proof content

(1) Set $\psi(\lambda) = \log \int e^{\lambda F} \, \mathrm{d}\mu$ and prove

$$\lambda\psi'(\lambda) - \psi(\lambda) = \frac{\mathrm{Ent}_\mu(e^{\lambda F})}{\int e^{\lambda F} \, \mathrm{d}\mu}.$$

(2) Apply the logarithmic Sobolev inequality to $g = \lambda F$ and obtain a differential inequality for $\psi(\lambda)/\lambda$.

(3) Integrate from 0 to λ, using $\lim_{\lambda\to 0} \psi(\lambda)/\lambda = \int F \, \mathrm{d}\mu$.

(4) Apply exponential Markov inequality and optimize in λ.

(5) Explain the approximation needed for a Lipschitz but nonsmooth observable such as $F(x) = |x|$.

5. Required examples and interpretation

Apply the theorem to the Euclidean norm of a standard Gaussian vector and explain the thin-shell conclusion. Include another Lipschitz observable, such as a coordinate maximum or distance to a set. At theorem-map level, explain that logarithmic Sobolev inequalities also convert entropy production into exponential relative-entropy decay.

6. What may be taken as input

The Gaussian logarithmic Sobolev inequality may be taken as a theorem; its proof is not part of Round I. You may first work with smooth bounded F and then discuss truncation and approximation.

7. Exercises and common pitfalls

Exercises may ask for Chernoff optimization, the two-sided tail, or a shell bound. Do not confuse thermodynamic/Shannon entropy with the functional entropy $\mathrm{Ent}_\mu(h)$. Track C and L throughout. A Poincare inequality alone does not yield the same Gaussian tail by this argument.

8. Suggested reading route and attribution

(1) R. van Handel, *Probability in High Dimension*, Section 3.3, approximately pp. 55–61, for the entropy method and Herbst-type argument (public course notes).

(2) D. Chafaï and J. Lehec, *Logarithmic Sobolev Inequalities Essentials*, Section 1.3 and Section 3.1, approximately pp. 11–14 and 35–37 (public lecture notes).

(3) M. Ledoux, “Concentration of measure and logarithmic Sobolev inequalities,” *Seminaire de Probabilites XXXIII*, Lecture Notes in Mathematics 1709 (1999), 120–216, especially the Laplace-transform/Herbst argument. DOI; public EuDML copy.

(4) D. Chafaï, *Mini-course on logarithmic Sobolev inequalities*, 2024, especially Section 4 (public mini-course notes).

9. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

10. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I ALTERNATE TOPIC PACKET 08 FOURIER ANALYSIS, POINCARÉ INEQUALITY, AND INFLUENCES ON THE BOOLEAN CUBE

MATTHEW ROSENZWEIG

ABSTRACT. **Organizing question:** How does Fourier analysis convert coordinate sensitivity into a variance bound? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Alternate Round I topic
Setting	Discrete product space $\{-1,1\}^n$
Primary tools	Walsh–Fourier basis, Parseval, discrete derivatives, influences
Background	Linear algebra; finite probability; elementary combinatorics
Relative difficulty	3/4
Principal perspective	Discrete probability and theoretical computer science

1. Why this topic may appeal to you

Choose this alternate if you are interested in discrete mathematics, theoretical computer science, learning theory, or Boolean functions. It gives a finite-dimensional Fourier theory parallel to the cycle calculation but emphasizes coordinate sensitivity rather than geometry in physical space.

2. Fixed setting

Let $\Omega = \{-1,1\}^n$ with uniform measure. For $S \subseteq [n]$, define

$$\chi_S(x) = \prod_{i\in S} x_i, \qquad D_i f(x) = \frac{f(x) - f(x^{\oplus i})}{2},$$

where $x^{\oplus i}$ flips coordinate i.

3. Principal theorem

Develop

$$f = \sum_{S\subseteq[n]} \widehat{f}(S)\chi_S$$

Date: Fall 2026.

and prove

$$\mathrm{Var}(f) = \sum_{S \neq \varnothing} \widehat{f}(S)^2, \qquad \mathbf{E}[(D_i f)^2] = \sum_{S \ni i} \widehat{f}(S)^2.$$

Conclude

$$\mathrm{Var}(f) \leq \sum_{i=1}^{n} \mathbf{E}[(D_i f)^2].$$

For $f : \Omega \to \{-1, 1\}$, show

$$\mathbf{E}[(D_i f)^2] = \mathbf{P}(f(X) \neq f(X^{\oplus i})) =: \mathrm{Inf}_i(f).$$

4. Required proof content

Prove orthonormality of Walsh characters, Fourier inversion, and Parseval. Compute $D_i \chi_S$ and derive the influence identity. Identify equality cases in the Poincare inequality.

5. Applications, exercises, and pitfalls

Compute the total influence of dictators and parity and analyze majority in small dimensions. Explain the conclusion for a balanced Boolean function. Exercises may ask for the Fourier expansion of a two-bit function or influence computations. Do not mix $\{0, 1\}$ and $\{-1, 1\}$ conventions or derivative normalizations.

6. What may be taken as input

You may use finite-dimensional orthogonal expansion once the basis and normalization have been established. Hypercontractivity is not part of this assignment.

7. Suggested reading route and attribution

(1) R. O'Donnell, *Analysis of Boolean Functions*, Chapter 1 for Fourier expansion and Chapter 2, especially Sections 2.2–2.4, for influences and basic inequalities. Public author copy; arXiv edition.
(2) R. van Handel, *Probability in High Dimension*, Section 2.1 for the product-space variance viewpoint (public course notes).

8. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

9. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY
ROUND I ALTERNATE TOPIC PACKET 09
PINSKER INEQUALITY AND ENTROPY-TO-TOTAL-VARIATION CONVERGENCE

MATTHEW ROSENZWEIG

ABSTRACT. **Organizing question:** How does relative entropy control probabilistic distinguishability? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

AT A GLANCE

Status	Alternate Round I topic
Setting	Information theory on finite probability spaces
Primary tools	Relative entropy, log-sum inequality, coarse-graining, convexity
Background	Finite probability; single-variable calculus
Relative difficulty	2/4
Principal perspective	Information theory and convergence to equilibrium

1. Why this topic may appeal to you

This alternate is a focused information-theoretic topic with a complete elementary proof. It explains how an analytic entropy estimate becomes an operational probabilistic statement about events and mixing.

2. Fixed conventions

For probability measures ν, μ on a finite set with $\mu(x) > 0$, use natural logarithms and define

$$D(\nu\|\mu) = \sum_x \nu(x) \log \frac{\nu(x)}{\mu(x)}, \qquad \|\nu - \mu\|_{\mathrm{TV}} = \frac{1}{2} \sum_x |\nu(x) - \mu(x)|.$$

3. Principal theorem

Prove Pinsker's inequality

$$\|\nu - \mu\|_{\mathrm{TV}} \leq \sqrt{\frac{1}{2} D(\nu\|\mu)}.$$

Then deduce from

$$D(\nu_t\|\mu) \leq e^{-\alpha t} D(\nu_0\|\mu)$$

Date: Fall 2026.

that

$$\|\nu_t - \mu\|_{\mathrm{TV}} \le \frac{1}{\sqrt{2}} e^{-\alpha t/2} \sqrt{D(\nu_0\|\mu)}.$$

4. Required proof route

(1) Prove the log-sum inequality and use it to show relative entropy decreases under a two-set coarse-graining.
(2) Let $A = \{x : \nu(x) \ge \mu(x)\}$ and prove $\nu(A) - \mu(A) = \|\nu - \mu\|_{\mathrm{TV}}$.
(3) Reduce to Bernoulli parameters $p = \nu(A)$ and $q = \mu(A)$.
(4) By Taylor's theorem and $[p(1-p)]^{-1} \ge 4$, prove

$$p \log \frac{p}{q} + (1-p) \log \frac{1-p}{1-q} \ge 2(p-q)^2.$$

5. Interpretation, exercises, and pitfalls

Explain total variation as the maximal discrepancy on an event and relative entropy as a nonsymmetric information discrepancy. Exercises may check the two-point case, prove the event characterization of total variation, or translate an entropy-decay rate into a mixing estimate. The orientation of $D(\nu\|\mu)$ matters; Pinsker is one-way; and changing the $1/2$ convention in total variation changes the constant.

6. Suggested reading route and attribution

(1) R. van Handel, *Probability in High Dimension*, Section 4.1, especially the discussion around pp. 80–82 of Pinsker and transportation-type inequalities (public course notes).
(2) I. Csiszar, "Information-type measures of difference of probability distributions and indirect observations," *Studia Scientiarum Mathematicarum Hungarica* 2 (1967), 299–318, for the broader divergence framework and data-processing viewpoint.

7. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

8. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I ALTERNATE TOPIC PACKET 10 ENTROPY, LIKELIHOOD, AND GIBBS EQUILIBRIUM

MATTHEW ROSENZWEIG

Abstract. **Organizing question:** Why does relative entropy appear in both statistical inference and equilibrium statistical mechanics? This packet fixes the mathematical scope and normalization, identifies the required proof content, and provides a curated reading route. It is intended both for informed preference ranking and for preparation after assignment.

At a glance

Status	Alternate Round I topic
Setting	Finite probability spaces
Primary tools	Jensen, relative entropy, likelihood identities, variational principles
Background	Finite probability; logarithms; elementary optimization
Relative difficulty	2/4
Principal perspective	Statistical/information-theoretic and physical

1. Why this topic may appeal to you

This alternate is the most conceptual bridge between statistics and physics. It is not a broad historical survey: the talk is organized around two exact identities showing that the same relative entropy measures likelihood loss and free-energy excess.

2. Statistical identity

Let $x_1, \dots, x_n$ lie in a finite alphabet, let $\widehat{p}_n$ be the empirical law, and let q be positive on the observed symbols. Prove

$$\frac{1}{n} \log \prod_{j=1}^{n} q(x_j) = \sum_x \widehat{p}_n(x) \log q(x) = -H(\widehat{p}_n) - D(\widehat{p}_n \| q),$$

where $H(p) = -\sum_x p(x) \log p(x)$. Conclude that the maximum-likelihood estimate over all probability laws is $q = \widehat{p}_n$. Also prove

$$\mathbf{E}_p \left[\log \frac{q(X)}{p(X)} \right] = -D(p\|q).$$

Date: Fall 2026.

3. Physical identity

Let $E(x)$ be an energy and $\beta > 0$. Define

$$\mu_\beta(x) = \frac{e^{-\beta E(x)}}{Z_\beta}, \qquad \mathcal{F}_\beta(\nu) = \sum_x E(x)\nu(x) + \frac{1}{\beta}\sum_x \nu(x)\log\nu(x).$$

Prove

$$\mathcal{F}_\beta(\nu) - \mathcal{F}_\beta(\mu_\beta) = \frac{1}{\beta} D(\nu\|\mu_\beta),$$

and conclude that μ_β uniquely minimizes free energy.

4. Required proof content

Prove nonnegativity and the equality case for relative entropy using Jensen or the log-sum inequality. Derive both identities line by line. Explain why reversing the two arguments of relative entropy changes the interpretation.

5. Optional extension, exercises, and pitfalls

If time permits, use Stirling's formula to explain why the number of strings of empirical type p grows like $e^{nH(p)}$. Exercises may treat a two-level Gibbs system, Bernoulli maximum likelihood, or a three-state free-energy calculation. State that Boltzmann's constant has been set to one. Do not call relative entropy a metric, and distinguish thermodynamic entropy from relative entropy.

6. Suggested reading route and attribution

(1) D. Chafaï, *Mini-course on logarithmic Sobolev inequalities*, Sections 1–2, approximately pp. 1–3, for entropy in combinatorics, probability and statistics and the Boltzmann–Gibbs/free-energy identities (public mini-course notes).

(2) T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed., Wiley, 2006, Chapter 2 for entropy and relative entropy and Chapter 11 for the method of types.

(3) R. van Handel, *Probability in High Dimension*, Section 4.1 for variational and information-theoretic context (public course notes).

7. Before the required consultation

Bring a completed one-page preparation document containing: the exact theorem statements; a dependency map of the proof; the portion you will prove in full; planned examples; two exercises with solutions; exact source locations; anticipated questions; and any point you do not yet understand. The consultation is for resolving mathematical and expository issues, not for choosing a topic from scratch.

8. AI-use disclosure

The preparation document must disclose any use of generative artificial-intelligence tools. Identify the tool, the task for which it was used, material incorporated into the presentation, and how mathematical assertions and references were independently verified. You remain responsible for every statement in the talk.

This is an instructor-authored assignment guide, not a substitute for reading the listed sources. All normalizations in the talk should agree with this packet unless a change is explicitly approved in advance.

21-317 FUNCTIONAL INEQUALITIES: FROM ANALYSIS TO PROBABILITY ROUND I ONLINE PREFERENCE SUBMISSION

MATTHEW ROSENZWEIG

Deadline: Sunday, August 30, 2026, at 11:59 p.m. EDT

Submit your ranked Round I topic preferences through the Google Form linked in the August 28 course announcement. The announcement is also accessible from the course website. Preferences sent by email will not count as a form submission.

Before submitting, read the topic catalogue, browse the first-page overview of every principal packet, and read the complete packets for your likely top choices. The form asks you to provide:

- five distinct ranked topic choices;
- at most one topic you strongly prefer not to receive;
- relevant mathematical background and areas of interest;
- genuine scheduling conflicts among the anticipated Round I dates; and
- your current presentation confidence and any especially useful form of instructor support.

Assignments are **not first-come-first-served**. They will balance student preferences, preparation, presentation order, and mathematical coverage. You may edit a submitted response until the deadline using the edit-response link supplied by Google Forms.

The anticipated Round I dates are September 14, 16, 18, 21, 23, 25, and 28. List only genuine conflicts.

If a technical problem prevents you from accessing or submitting the form, contact the instructor before the deadline. As required by the course email policy, begin the subject line with `[21-317]`.

Date: Fall 2026.