21-317: Suggested Research Papers

Final project, Fall 2026

Revised October 23, 2026

Suggestions, not a required or exhaustive list. You may propose another original research paper after consulting the instructor. Choose a subject because the mathematical question interests you. Attempt to understand the entire paper as fully as possible, then explain its main theorem, its proof strategy and a substantial argument that you have worked through yourself.

Selection. Review several options. A first-come, first-served form opens **Tuesday, October 27**. Individual suggestions connected to Round I will also be sent by email.

Presentations. November 18, 20, 23, and 30; two presentations per meeting, **25 minutes each**.

Expository write-up. Approximately **10 pages**, due **Friday, December 4**. For a very short article, expand compressed arguments and supply useful background. For a longer article, distill its essential mathematics without claiming to prove everything.

Schedule. November 25 and 27 are Thanksgiving break; no class December 2 or 4 so you have time to work on the written paper.

Difficulty (entire original). A: modest additional preparation; **B**: substantial but manageable prerequisites; **C**: ambitious, requiring advance discussion with the instructor. Intermediate ratings depend strongly on your background; a very short paper need not be easy.

If you are unsure where to begin: Loomis–Whitney, Hoeffding, Efron–Stein, Dasgupta–Gupta, Hastings, and Rahimi–Recht are possible starting points. The Bauerschmidt–Bodineau spin-system paper is our shared classroom reading rather than a separate project option.

1. Functional inequalities, geometry, and analysis

Pure analysis: how inequalities capture geometry, regularity, and the behavior of functions.

Loomis–Whitney (1949). *An inequality related to the isoperimetric inequality.* **[A; 2 pp.]**
Bull. Amer. Math. Soc. 55, 961–962. Paper or publisher record

- *Why this paper?* A foundational inequality comparing the volume of a set with the volumes of its coordinate projections.
- *Mathematical focus.* Reconstruct the original discrete counting inequality (Theorem 2), then derive the volume inequality (Theorem 1).
- *Preparation / limitations.* Finite sums, elementary geometric measure, counting; a natural first paper.

Payne–Weinberger (1960). *An optimal Poincaré inequality for convex domains.* **[B/C; 7 pp.]**
Arch. Ration. Mech. Anal. 5, 286–292. Paper or publisher record

- *Why this paper?* A sharp diameter-dependent Poincaré inequality on convex domains.
- *Mathematical focus.* Understand the one-dimensional inequality and trace how convex geometry enters the higher-dimensional argument.
- *Preparation / limitations.* One-dimensional Poincaré inequalities, calculus and convex geometry. The original proof treats planar convex domains; the higher-dimensional extension requires care.

Beckner (1989). *A generalized Poincaré inequality for Gaussian measures.* **[B/C; 4 pp.]**
Proc. Amer. Math. Soc. 105, 397–400. Paper or publisher record

- *Why this paper?* Sharp inequalities interpolating between Gaussian Poincaré and logarithmic Sobolev inequalities.

- *Mathematical focus.* Explain the parameter family, recover both endpoints, and investigate the proof of the interpolation inequality.
- *Preparation / limitations.* Gaussian integration and semigroup or Hermite-polynomial methods. The four-page original is very compressed.

Talagrand (1996). *Transportation cost for Gaussian and other product measures.* **[B/C; 14 pp.]**
Geom. Funct. Anal. 6, 587–600. Paper or publisher record

- *Why this paper?* A foundational quadratic transportation-cost inequality for Gaussian and product probability measures.
- *Mathematical focus.* Understand the one-dimensional Gaussian transport inequality, its entropy bound, and the role of product measures.
- *Preparation / limitations.* Probability densities, entropy and monotone rearrangement. The full argument is more demanding.

Bobkov (1997). *An isoperimetric inequality on the discrete cube, and an elementary proof of the isoperimetric inequality in Gauss space.* **[C; 9 pp.]**
Ann. Probab. 25, 206–214. Paper or publisher record

- *Why this paper?* An original functional inequality on the discrete cube leading to Gaussian isoperimetry.
- *Mathematical focus.* Start with the basic two-point inequality, then show how tensorization produces a discrete isoperimetric statement.
- *Preparation / limitations.* Gaussian distributions, convexity and a nontrivial limiting argument.

Carlen–Loss (1993). *Sharp constant in Nash's inequality.* **[C; 3 pp.]**
Int. Math. Res. Notices 1993, 213–215. Paper or publisher record

- *Why this paper?* Determines the sharp constant in the Nash inequality.
- *Mathematical focus.* Explain Nash's inequality and why sharpness matters; reconstruct the optimization and symmetrization steps.
- *Preparation / limitations.* Sobolev inequalities, variational calculus and rearrangement. Short but not elementary.

Gross (1975). *Logarithmic Sobolev inequalities.* **[C; 23 pp.]**
Amer. J. Math. 97, 1061–1083. Paper or publisher record

- *Why this paper?* The foundational development of Gaussian logarithmic Sobolev inequalities and hypercontractivity.
- *Mathematical focus.* Prove a finite-dimensional Gaussian consequence, then explain the equivalence with hypercontractivity and how the paper proceeds beyond it.
- *Preparation / limitations.* Semigroups, functional analysis and some infinite-dimensional arguments.

Jordan–Kinderlehrer–Otto (1998). *The variational formulation of the Fokker–Planck equation.* **[C; 17 pp.]**
SIAM J. Math. Anal. 29, 1–17. Paper or publisher record

- *Why this paper?* A foundational connection between free-energy dissipation and optimal transport.
- *Mathematical focus.* Derive the implicit variational time step in a simple model, and explain its relation to diffusion and entropy.
- *Preparation / limitations.* Calculus of variations, probability densities, PDE and Wasserstein distance.

2. Concentration and high-dimensional probability

Concentration and variance inequalities for random functions, including high-dimensional examples.

Hoeffding (1963). *Probability inequalities for sums of bounded random variables.* **[B; 18 pp.]**
J. Amer. Statist. Assoc. 58, 13–30. Paper or publisher record

- *Why this paper?* Classical exponential concentration bounds.
- *Mathematical focus.* Prove an exponential-moment estimate, derive a bounded-sum tail inequality, then account for the sampling-without-replacement results.
- *Preparation / limitations.* Independent random variables, exponential moments and convexity.

Efron–Stein (1981). *The jackknife estimate of variance.* **[B; 11 pp.]**
Ann. Statist. 9, 586–596. Paper or publisher record

- *Why this paper?* A foundational comparison of the jackknife variance estimator with actual variance.
- *Mathematical focus.* Recover the orthogonal decomposition of variance and derive the principal jackknife estimate.
- *Preparation / limitations.* Conditional expectation and the variance of jackknife estimates; do not confuse the original paper with only the modern one-line Efron–Stein inequality.

Dasgupta–Gupta (2003). *An elementary proof of a theorem of Johnson and Lindenstrauss.* **[A/B; 6 pp.]**
Random Structures Algorithms 22, 60–65. Paper or publisher record

- *Why this paper?* An influential, especially readable later proof of the original Johnson–Lindenstrauss dimension-reduction theorem.
- *Mathematical focus.* Analyze random projections, obtain concentration of lengths, then take a union bound over pairs of points.
- *Preparation / limitations.* Gaussian probability, elementary exponential-moment estimates and union bounds. This is a later proof, not the original discovery.

3. Entropy and discrete inequalities

Entropy and information as tools for proving combinatorial and analytic inequalities.

Kahn (2002). *Entropy, independent sets and antichains: a new approach to Dedekind's problem.* **[B/C; 8 pp.]**
Proc. Amer. Math. Soc. 130, 371–378. Paper or publisher record

- *Why this paper?* Entropy inequalities are a key ingredient in counting independent sets and antichains.
- *Mathematical focus.* Identify the precise entropy estimate and reconstruct its use in bounding the number of independent sets; relate this to the hard-core partition function.
- *Preparation / limitations.* Discrete entropy, conditional probability, finite combinatorics; more directly inequality-based than Zhao (2010).

Lovász (1979). *On the Shannon capacity of a graph.* **[B/C; 7 pp.]**
IEEE Trans. Inform. Theory 25, 1–7. Paper or publisher record

- *Why this paper?* The theta-function method for bounding zero-error communication capacity.
- *Mathematical focus.* Explain the orthogonal-representation bound and the exact calculation for the pentagon.
- *Preparation / limitations.* Linear algebra, graph theory and positive semidefinite matrices.

Madiman–Tetali (2010). *Information inequalities for joint distributions, with interpretations and applications.* **[C; 15 pp.]**
IEEE Trans. Inform. Theory 56, 2699–2713. Paper or publisher record

- *Why this paper?* Refined entropy inequalities using fractional coverings and submodularity.
- *Mathematical focus.* Prove one fractional-cover inequality, then describe the other applications to joint distributions.
- *Preparation / limitations.* Conditional entropy, combinatorics of set systems.

Stam (1959). *Some inequalities satisfied by the quantities of information of Fisher and Shannon.* **[C; 12 pp.]**
Information and Control 2, 101–112. Paper or publisher record

- *Why this paper?* Foundational Fisher-information and entropy-power inequalities.
- *Mathematical focus.* Isolate the Fisher-information inequality and explain its connection with convolution and Gaussian noise.
- *Preparation / limitations.* Probability densities, differentiation and convolution; some proofs in the original are not fully elaborated.

4. Markov chains, spectral gaps, and sampling

Convergence to equilibrium, its relation to the spectrum, and exact or approximate sampling.

Hastings (1970). *Monte Carlo sampling methods using Markov chains and their applications.* **[B; 13 pp.]**
Biometrika 57, 97–109. Paper or publisher record

- *Why this paper?* Generalizes Metropolis sampling to asymmetric proposals and studies Monte Carlo error.
- *Mathematical focus.* Derive the acceptance rule and detailed balance; discuss why serial correlation affects estimates.
- *Preparation / limitations.* Reversible chains, conditional probability and variance.

Diaconis–Stroock (1991). *Geometric bounds for eigenvalues of Markov chains.* **[B/C; 26 pp.]**
Ann. Appl. Probab. 1, 36–61. Paper or publisher record

- *Why this paper?* The canonical-path method relating graph geometry to spectral gaps.
- *Mathematical focus.* Derive Proposition 1 (the canonical-path Poincaré bound); apply it to the cycle, then explain the other eigenvalue estimates.
- *Preparation / limitations.* Finite reversible chains, Dirichlet forms, and the Rayleigh quotient. A substantial full-paper project despite the elementary central proof.

Propp–Wilson (1996). *Exact sampling with coupled Markov chains and applications to statistical mechanics.* **[C; 30 pp.]**
Random Structures Algorithms 9, 223–252. Paper or publisher record

- *Why this paper?* Introduces coupling from the past for exact sampling from stationary laws.
- *Mathematical focus.* Prove exactness in a finite monotone-chain example and account for the larger applications.
- *Preparation / limitations.* Coupling, Markov chains and partial orders; this is a long project.

Diaconis–Saloff-Coste (2006). *Separation cut-offs for birth and death chains.* **[C; 25 pp.]**
Ann. Appl. Probab. 16, 2098–2122. Paper or publisher record

- *Why this paper?* A theorem connecting the cutoff phenomenon to the spectral gap and a spectral time scale for birth-and-death chains.
- *Mathematical focus.* Begin with the precise cutoff criterion in Theorem 1.1 and examples; identify the role of eigenvalues and explain which spectral and hitting-time results the proof requires.
- *Preparation / limitations.* Finite-state continuous-time Markov chains, spectral gaps, hitting times. The result concerns separation from an endpoint, not arbitrary chains or all distances.

5. Correlation inequalities and mathematical statistical mechanics

What inequalities imply about correlations, monotonicity, and equilibrium in interacting systems.

Fortuin–Kasteleyn–Ginibre (1971). *Correlation inequalities on some partially ordered sets.* **[B/C; 15 pp.]**
Commun. Math. Phys. 22, 89–103. Paper or publisher record

- *Why this paper?* The original FKG positive-association theorem for measures on finite distributive lattices.
- *Mathematical focus.* State Proposition 1 (positive association under the lattice condition), examine the induction, and work out an attractive Ising example.
- *Preparation / limitations.* Finite probability, monotone functions and lattice operations. The induction needs close reading; the Ising application is more approachable.

Griffiths–Hurst–Sherman (1970). *Concavity of magnetization of an Ising ferromagnet in a positive external field.* **[C; 6 pp.]**
J. Math. Phys. 11, 790–795. Paper or publisher record

- *Why this paper?* A classical higher-order correlation inequality and magnetization concavity.
- *Mathematical focus.* Identify the sign inequality and explain how it yields concavity in the applied field.
- *Preparation / limitations.* Ising systems, magnetization derivatives and correlation inequalities. Particularly demanding in its short format.

Edwards–Sokal (1988). *Generalization of the Fortuin–Kasteleyn–Swendsen–Wang representation and Monte Carlo algorithm.* **[B/C; 4 pp.]**
Phys. Rev. D 38, 2009–2012. Paper or publisher record

- *Why this paper?* An influential joint spin–bond representation connecting Potts models and cluster sampling.
- *Mathematical focus.* Check the spin–bond coupling on a small graph and derive the conditional spin-cluster law.
- *Preparation / limitations.* Finite graph probability and Gibbs measures; it is a representation identity, not itself a functional inequality.

Holley–Stroock (1987). *Logarithmic Sobolev inequalities and stochastic Ising models.* **[C; 36 pp.]**
J. Stat. Phys. 46, 1159–1194. Paper or publisher record

- *Why this paper?* A foundational application of logarithmic Sobolev inequalities to stochastic Ising dynamics.
- *Mathematical focus.* Identify where logarithmic Sobolev inequalities control the approach to equilibrium for stochastic Ising systems; examine a finite example.
- *Preparation / limitations.* Markov semigroups, conditional entropy and spin-flip dynamics. A longer, technical project.

6. Statistical learning and information inequalities

Original papers in which a precise probability, entropy, or stability inequality is a principal mathematical result.

McAllester (1998). *Some PAC-Bayesian theorems.* **[B/C; 5 pp.]**
Proc. 11th COLT, 230–234. Paper or publisher record

- *Why this paper?* An early foundational PAC-Bayesian generalization inequality based on relative entropy.
- *Mathematical focus.* Work out the exponential change-of-measure inequality between a prior and posterior, then reconstruct a PAC-Bayesian bound.
- *Preparation / limitations.* Hoeffding-type concentration, relative entropy and conditional probability. The original is only five pages and very compressed.

Rahimi–Recht (2007). *Random features for large-scale kernel machines.* **[B; 8 pp.]**
Adv. Neural Inf. Process. Syst. 20, 1177–1184. Paper or publisher record

- *Why this paper?* Random Fourier features with quantitative concentration bounds for approximating kernels.
- *Mathematical focus.* Verify the random-feature expectation identity, the fixed-pair concentration bound, and the passage toward uniform approximation.
- *Preparation / limitations.* Bounded-variable concentration and basic Fourier analysis; Bochner's theorem may be quoted.

Hardt–Recht–Singer (2016). *Train faster, generalize better: stability of stochastic gradient descent.* **[B; 10 pp.]**
Proc. ICML (PMLR 48), 1225–1234. Paper or publisher record

- *Why this paper?* Quantitative stability estimates for stochastic gradient descent and their implications for generalization.
- *Mathematical focus.* Reconstruct the stability-to-generalization estimate and the contraction bounds for convex gradient updates.
- *Preparation / limitations.* Lipschitz/convex functions, iterative algorithms, probability. The connection to this course is quantitative stability, not a logarithmic Sobolev inequality.

Lee–Lu–Tan (2022). *Convergence for score-based generative modeling with polynomial complexity.* **[C; 13 pp.]**
Adv. Neural Inf. Process. Syst. 35, 22870–22882. Paper or publisher record

- *Why this paper?* Quantitative convergence guarantees for score-based generative sampling, using logarithmic Sobolev inequalities to control sampling with an approximate score.
- *Mathematical focus.* Understand Theorem 2.1 and prove the coupling estimate in Theorem 4.1 (Section 4); explain how the log-Sobolev hypothesis enters the later χ^2 estimate.
- *Preparation / limitations.* Relative entropy, χ^2 divergence, Markov chains and Langevin diffusion. The main text is 13 pages, but full proofs are substantially longer; this is an ambitious project requiring instructor guidance.