

Functional Inequalities: From Analysis to Probability

Topics in Analysis (21-317)

Fall 2026

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Lectures: MWF, 12:00–12:50 PM; WEH 7201
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1. INTRODUCTION

Functional inequalities quantify a recurring local-to-global principle: control of a function through a local energy, gradient, boundary, or dissipation term can force global control of its fluctuations. This course is an introduction to several central examples in modern analysis and probability, including Poincaré, Sobolev, logarithmic Sobolev, Nash, isoperimetric, and Cheeger-type inequalities, together with selected applications. Topics will include spectral gaps and variance decay, concentration of measure, entropy methods, heat-flow smoothing, and convergence to equilibrium for Markov processes. Examples will be drawn from Euclidean, Gaussian, finite-state, graph, and other discrete settings, with selected applications to partial differential equations, mathematical physics, statistics, and machine learning.

The course is also designed to develop students' ability to read advanced mathematics, explain it accurately, respond thoughtfully to questions, and write about it effectively. The opening part of the semester will consist primarily of instructor-led lectures establishing a common foundation. Much of the remainder will be organised around student presentations of progressively more advanced material. Students will prepare each presentation in consultation with the instructor, design short exercises for the class, receive structured feedback, and complete an expository paper on a topic related to the course.

Throughout the semester, we will return to several complementary perspectives: analytic and dynamical, physical, statistical and information-theoretic, and expository. These perspectives will emerge through examples rather than be imposed as a rigid taxonomy.

Prerequisites and expected background. Students should have prior experience with proof-based analysis and some background in probability. The required prerequisites are

(21-355 with minimum grade A, or 21-235) and (21-325, 15-259, 36-218, or 36-225).

The course 21-356 is recommended. Further coursework in analysis or probability, such as 21-720 or 21-326, may be helpful but is not necessary. Measure theory, functional analysis, distribution theory, and Sobolev spaces are not assumed; the portions needed for the course will be supplied as they arise. Students should nevertheless expect to fill occasional gaps in their background through independent reading.

2. LEARNING OBJECTIVES

By the end of the course, students should be able to:

- state and interpret the principal functional inequalities studied in the course in representative continuous and discrete settings;
- prove foundational examples using tools such as linear algebra, Fourier series, variational arguments, semigroup identities, conditioning, and tensorisation;

- explain consequences for spectral gaps, concentration, smoothing, mixing, and convergence to equilibrium;
- explain the roles of entropy and relative entropy in probability, statistical mechanics, information theory, and dissipative dynamics;
- read advanced mathematical sources strategically, identifying hypotheses, dependencies, proof mechanisms, and invoked background results;
- present mathematics accurately and responsively, including adapting to questions and being explicit about the limits of one's understanding; and
- write a coherent expository mathematical paper with precise statements, meaningful proof content, examples, and proper attribution.

3. TENTATIVE MATHEMATICAL CONTENT

The exact selection and order of later topics will depend partly on student interests and background. The common core will include most of the following.

Foundations. Finite probability spaces; expectation, variance, entropy, and relative entropy; Gibbs measures and free energy; reversible finite-state Markov chains; Dirichlet forms; selected elements of Lebesgue integration; weak derivatives and Sobolev spaces.

Core inequalities. Poincaré and spectral-gap inequalities; Sobolev, Nash, isoperimetric, and Cheeger-type inequalities; logarithmic Sobolev inequalities; examples in Euclidean, Gaussian, compact, graph, product, and other discrete settings.

Consequences and mechanisms. Variance and entropy decay; concentration of measure; heat-flow smoothing; mixing and convergence to equilibrium; Fourier and spectral methods; semigroup interpolation; tensorisation and conditioning; convexity, comparison, and perturbation principles; the relation between geometry, bottlenecks, and coercive constants.

Selected modern topics. Depending on student interest, later presentations may concern high-dimensional geometry, Markov-chain mixing, Langevin sampling, spin systems, interacting-particle systems, optimal transport, nonlinear diffusion, theoretical computer science, statistics, or machine learning.

4. COURSE MATERIALS AND REFERENCES

There is no required textbook and no single source is tailored to the background and aims of this course. Readings will be assigned selectively, usually with precise page or section ranges, and additional references will be recommended according to presentation and paper topics. Particularly useful sources include:

- D. Chafaï and J. Lehec, *Logarithmic Sobolev Inequalities Essentials*, public lecture notes: <https://djalil.chafai.net/docs/M2/chafai-lehec-m2-lsie-lecture-notes.pdf>.
- D. Chafaï, *Mini-course on Logarithmic Sobolev Inequalities* (2024), public lecture notes: <https://djalil.chafai.net/docs/chafai-mini-lsi-2024.pdf>.
- D. A. Levin and Y. Peres, with contributions by E. L. Wilmer, *Markov Chains and Mixing Times*, 2nd ed., public author-hosted PDF: <https://pages.uoregon.edu/dlevin/MARKOV/mcmt2e.pdf>.
- M. Ledoux, *Concentration of Measure and Logarithmic Sobolev Inequalities*, public lecture notes: <https://www.math.univ-toulouse.fr/~ledoux/Berlin.pdf>.
- D. Bakry, I. Gentil, and M. Ledoux, *Analysis and Geometry of Markov Diffusion Operators*, Springer, 2014.
- For analytic background, selected portions of L. C. Evans, *Partial Differential Equations*, 2nd ed., and E. H. Lieb and M. Loss, *Analysis*, 2nd ed., may be useful.

Course materials supplied by the instructor will be posted on Canvas. Students should not assume that every source listed above is intended to be read from beginning to end.

5. COURSE FORMAT AND ASSESSMENT

There are no examinations and no problem sets. Course performance will be evaluated holistically on the basis of student presentations, the expository paper and its milestones, attendance and engagement, demonstrated mathematical understanding and effort, receptiveness to feedback, and improvement over the semester. There are no fixed numerical weights for these components, and individual presentations will not receive separate letter or numerical grades.

Timely and substantive completion matters. In particular, a placeholder submission does not count as completion of a presentation-preparation document or paper milestone. The preliminary paper milestones are evaluated principally on completion and usefulness for the revision process; the revised final paper is evaluated for both mathematical and expository quality.

Student presentations. The current plan, subject to adjustment after enrolment stabilises, is approximately four progressively more demanding presentation rounds:

- (1) a foundational theorem, examples, and one complete proof;
- (2) a proof mechanism, structural equivalence, or comparison principle;
- (3) a substantial application or extension, including a significant proof component; and
- (4) a synthesis of a modern topic using several sources, potentially including selected portions of an original research paper.

The progression concerns the difficulty and scope of the expository task. Students may change mathematical topics between rounds.

Each student must schedule an individual consultation with the instructor before each presentation and submit the required preparation document in advance of that meeting. The preparation document will include the principal theorem, its hypotheses, a proof-dependency map, the portion to be proved in detail, background required by the audience, exercises with solutions, precise source locations, and anticipated questions.

Questions from the instructor and other students are an integral part of every presentation, not interruptions. Presenters should expect to pause, clarify definitions, justify hypotheses, produce examples, or modify their allocation of time. It is acceptable—and preferable—to say “I do not know” rather than to improvise an unsupported answer. Feedback will address mathematical correctness and precision, depth of understanding, organisation, clarity, audience awareness, source use and attribution, handling of questions, time management, and incorporation of earlier feedback.

Each presenter will pose a small number of short exercises related to the talk. Students are expected and encouraged to work on these exercises, individually or collaboratively, but solutions will not be collected or graded.

For each presentation, two students will serve as designated peer respondents. Their feedback will be named, structured, and submitted privately to the presenter and instructor. Peer feedback is formative and does not determine the presenter’s evaluation; giving thoughtful feedback is itself part of course engagement.

Expository paper. Each student will write an expository paper of approximately 10–15 pages, excluding the bibliography, on a topic connected to the course. Mathematical substance matters more than page count. The paper should contain a clearly formulated theme, sufficient background, precise theorem statements and hypotheses, at least one substantial proof or proof component, examples or applications, a discussion of how the material fits into the broader course, and complete attribution of sources and proof strategies.

The paper may develop material from a presentation, but it should not be a transcript of the talk. The oral and written formats require different choices of scope, pacing, detail, and explanation.

The paper will be developed through required milestones: a topic proposal and preliminary sources, an annotated bibliography, a mathematical outline and theorem map, a partial draft, a complete draft, and a revised final submission. The dates will be announced separately on Canvas. Failure to complete milestones in a timely and substantive manner prevents meaningful feedback and will affect the holistic course evaluation.

Engagement. Constructive engagement can take several forms: asking a useful question, answering or extending another student's question, identifying a missing hypothesis, offering an example or counter-example, working on the presentation exercises, making a connection to earlier material, or providing thoughtful written peer feedback. Participation is not measured by speed of response or frequency of speaking alone.

6. ATTENDANCE

Attendance is required. The course cannot function without a consistent and attentive audience: questions and discussion are part of the mathematical content, designated respondents have responsibilities to the presenter, and failing to attend classmates' talks deprives them of the audience on which the seminar format depends.

Each student is allotted **three no-questions-asked excused absences** during the semester. These absences may not be used to avoid a scheduled presentation. If a genuine circumstance prevents a student from presenting as scheduled, the student should contact the instructor as soon as reasonably possible so that the talk can be rescheduled. Additional absences will be handled as circumstances arise. Persistent nonattendance, especially without communication, places a student at risk of not passing the course.

Students who miss class remain responsible for material, announcements, exercises, and changes to the schedule.

7. COLLABORATION, ATTRIBUTION, AND ARTIFICIAL INTELLIGENCE

Collaboration. Collaboration is encouraged when preparing presentations, discussing paper topics, rehearsing talks, locating sources, or working on exercises. Students may exchange ideas and feedback, but each presenter or author remains individually responsible for understanding and defending every mathematical assertion in the submitted or presented work. Substantive assistance from classmates, faculty, tutors, software, or other sources must be acknowledged.

References and attribution. All mathematical sources must be cited wherever they are used. Whenever reasonably possible, citations should identify the relevant page, section, theorem, or proposition. Established arguments and proof strategies must be attributed to their sources, even when rewritten in the student's own words. An artificial-intelligence system is not an adequate mathematical citation: claims first encountered through such a system must be verified against an appropriate source.

Artificial intelligence. Use of generative artificial-intelligence tools is permitted, provided that it is disclosed and properly attributed. An AI-use disclosure is required with each presentation, each paper submission or milestone, and any other submitted assignment. The disclosure should identify the tool, explain the purposes for which it was used, describe any material incorporated into the work, and state how mathematical claims and references were independently verified. A student who used no such tools should say so.

Students are responsible for all content regardless of how it was produced. Uncritical regurgitation of generated material is strongly discouraged. A student must be able to explain and defend every assertion in a

presentation or paper; the ordinary questioning component of the course will make superficial reproduction without understanding readily apparent.

A suitable disclosure has the form:

I used [tool] for [purposes]. I incorporated [brief description, or “no generated text”]. I verified mathematical statements and references using [sources].

These course-specific permissions and requirements supplement Carnegie Mellon’s [Policy on Academic Integrity](#). Honesty, transparent attribution, and an accurate representation of one’s own understanding are essential.

8. PROVISIONAL CALENDAR

The calendar below assumes the current enrolment of seven students and four presentation rounds. It will be adjusted if enrolment changes. Speaker numbers are placeholders, and the eventual order will be rotated between rounds. Paper milestones will be announced separately. The university calendar lists 13 Monday, 13 Wednesday, and 13 Friday class meetings for Fall 2026.

Week	Date	Meeting
1	M Aug 24	Bridge 1: Panorama—why should a function be almost constant?
1	W Aug 26	Bridge 2: Finite probability, entropy, and Gibbs equilibrium
1	F Aug 28	Bridge 3: Finite reversible Markov chains and Dirichlet forms
2	M Aug 31	Bridge 4: Poincaré inequality, spectral gap, and variance decay
2	W Sep 2	Bridge 5: Measure and integration as needed
2	F Sep 4	Bridge 6: Weak derivatives, Sobolev spaces, and periodic Poincaré
3	M Sep 7	X — No class: Labor Day
3	W Sep 9	Bridge 7: Sobolev, isoperimetric, and Nash inequalities
3	F Sep 11	Bridge 8: Logarithmic Sobolev inequalities, entropy decay, tensorisation, and concentration
4	M Sep 14	Round I — Speaker 1
4	W Sep 16	Round I — Speaker 2
4	F Sep 18	Round I — Speaker 3
5	M Sep 21	Round I — Speaker 4
5	W Sep 23	Round I — Speaker 5
5	F Sep 25	Round I — Speaker 6
6	M Sep 28	Round I — Speaker 7
6	W Sep 30	Instructor synthesis and mathematical-reading workshop
6	F Oct 2	Round II — Speaker 3
7	M Oct 5	Round II — Speaker 4
7	W Oct 7	Round II — Speaker 5
7	F Oct 9	Mathematical-writing workshop
8	M Oct 12	X — No class: Fall Break
8	W Oct 14	X — No class: Fall Break
8	F Oct 16	X — No class: Fall Break
9	M Oct 19	Round II — Speaker 6
9	W Oct 21	Round II — Speaker 7
9	F Oct 23	Round II — Speaker 1
10	M Oct 26	Round II — Speaker 2
10	W Oct 28	Round III — Speaker 5
10	F Oct 30	Round III — Speaker 6
11	M Nov 2	Round III — Speaker 7
11	W Nov 4	Round III — Speaker 1

Week	Date	Meeting
11	F Nov 6	Round III — Speaker 2
12	M Nov 9	Round III — Speaker 3
12	W Nov 11	Round III — Speaker 4
12	F Nov 13	Instructor synthesis and expository-project workshop
13	M Nov 16	Round IV — Speaker 7
13	W Nov 18	Round IV — Speaker 1
13	F Nov 20	Round IV — Speaker 2
14	M Nov 23	Round IV — Speaker 3
14	W Nov 25	X — No class: Thanksgiving Break
14	F Nov 27	X — No class: Thanksgiving Break
15	M Nov 30	Round IV — Speaker 4
15	W Dec 2	Round IV — Speaker 5
15	F Dec 4	Round IV — Speaker 6; last day of classes

9. POLICIES AND UNIVERSITY RESOURCES

Email communication. All emails to the instructor concerning the course must begin with [21-317] in the subject line. Messages that do not follow this convention will not receive a response. Emails that are clearly AI-generated slop will likewise be ignored.

Recording and distribution. No student may record classroom activity without the instructor's express written consent, except pursuant to an approved accommodation. Course materials and recordings, when authorized, may not be distributed outside the course without permission.

Accommodations for students with disabilities. If you have a disability and an accommodations letter from the Office of Disability Resources, please discuss your accommodations and needs with me as early in the semester as possible. I will work with you to ensure that approved accommodations are provided appropriately. Students who suspect that they may benefit from accommodations but are not yet registered should contact the [Office of Disability Resources](#); the office is located at 4980 Margaret Morrison Street and may be reached at 412-268-7632.

Health, well-being, and academic support. All of us benefit from support during periods of academic or personal difficulty. Carnegie Mellon maintains a central [Student Support and Resources](#) page. Confidential mental-health support is available from [Counseling and Psychological Services \(CaPS\)](#) at 412-268-2922. The [Student Academic Success Center](#) provides academic coaching, communication support, tutoring, and other resources.

Inclusive learning environment. It is my intention that students from all backgrounds and perspectives be well served by this course and that the diversity students bring to the class be treated as a resource. Please let me know of ways to improve the effectiveness or accessibility of the course. Students are also encouraged to consult Carnegie Mellon's [2026–2027 Student Handbook](#) for university-wide policies, rights, responsibilities, and resources.