A computer-assisted upper bound for the three-dimensional Coulomb Lieb–Oxford constant

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18 August 2026

Certified bound

In the convention in which the arguments of the sharp Lieb–Oxford constant are ordered as (Riesz exponent, dimension), the finite certificate gives

$$\boxed{c_{\mathrm{LO}}(1,3) \leq \frac{1575524158829}{10^{12}} = 1.575524158829.}$$

The computational campaign used the reversed ordering and recorded the same statement as $c_{\mathrm{LO}}(3,1) \leq 1.575524158829$. This slightly improves the published upper bound 1.58 of Lewin, Lieb, and Seiringer [LLS22].

The result is a computer-assisted theorem relative to the analytic reduction and analytic estimates described below. It is not a numerical approximation: the accepted upper endpoint is the exact rational number displayed above.

What the finite certificate checks

The certificate refines the tail computation in the Lewin–Lieb–Seiringer comparison. An independent Ruby verifier reconstructed the finite evidence using exact rational arithmetic. In particular, it checked

- 90 high-region cells and 110,178 Bernstein boxes;
- exact abutting coverage, endpoint and curvature evidence, secant integrals, and the tail contribution;
- an exact rational upper bound for π, the aggregate integral, and the outward-rounded cube containing the final constant; and
- strict improvement over the previously certified baseline.

The accepted replay used Ruby 2.6.10 and eight verification jobs and took 12,430.08 seconds. A separate corruption suite passed 12 runs and 49 assertions with no failures, errors, or skips.

The SHA-256 digest of the accepted receipt is

`cb1775b060f95e760c1d6f60da8aeaa4`
`023f80eb93f0409efb789df430ce91d2.`

Public artifacts and replay

The replay bundle contains the complete 90-cell evidence used by the independent verifier, the frozen baseline, the verifier source, the accepted receipt and transcript, the execution records, and replay instructions. Its SHA-256 digest is

`51ab065cee57a92d6e4360df97f5d2a4`
`ca85e95065650f4dc1a1eb00c83fc029`.

For convenient inspection, the accepted receipt and verification transcript are also available separately.

Trust boundary

The exact finite layer is independently reconstructed from rational candidate data and certificate files; it does not import the numerical discovery code. The reduction from the finite computation to the Lieb–Oxford inequality uses the Lewin–Lieb–Seiringer analytic theorem, the shell formula and radial decomposition, the feasible-majorant estimate, the Bernstein convex-hull and semiconvex interpolation arguments, and the candidate-specific shell-tail lemma. These are source-checked analytic inputs rather than kernel-formalized proof objects. The Ruby runtime, standard library, operating system, and hardware are also part of the trusted computing base. Thus, "machine verified" here means exact verification of the finite certificate relative to this disclosed analytic and computational trust base; it does not mean an end-to-end Lean proof.

References

[LLS22] M. Lewin, E. H. Lieb, and R. Seiringer, "Improved Lieb–Oxford bound on the indirect and exchange energies," *Letters in Mathematical Physics* **112** (2022), Paper No. 92. doi:10.1007/s11005-022-01584-5; arXiv:2203.12473.